

# The analemma criterion: accidental quasi-satellites are indeed true quasi-satellites

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## ABSTRACT

In the Solar system, a quasi-satellite is an object that follows a heliocentric path with an orbital period that matches almost exactly with that of a host body (planetary or not). The trajectory is of such nature that, without being gravitationally attached, the value of the angular separation between host and quasi-satellite as seen from the Sun remains confined within relatively narrow limits for time-spans that exceed the length of the host’s sidereal orbital period. Here, we show that under these conditions, a quasi-satellite traces an analemma in the sky as observed from the host in a manner similar to that found for geosynchronous orbits. The analemmatic curve (figure-eight-, teardrop-, ellipse-shaped) results from the interplay between the tilt of the rotational axis of the host and the properties of the orbit of the quasi-satellite. The analemma criterion can be applied to identify true quasi-satellite dynamical behaviour using observational or synthetic astrometry and it is tested for several well-documented quasi-satellites. For the particular case of 15810 (1994 JR<sub>1</sub>), a putative accidental quasi-satellite of dwarf planet Pluto, we show explicitly that this object describes a complex analemmatic curve for several Plutonian sidereal periods, confirming its transient quasi-satellite status.

**Key words:** methods: numerical – celestial mechanics – minor planets, asteroids: general – minor planets, asteroids: individual: 15810 (1994 JR<sub>1</sub>) – minor planets, asteroids: individual: 63252 (2001 BL<sub>41</sub>).

## 1 INTRODUCTION

Objects trapped in a 1:1 mean motion resonance with a host (planetary or not) are classified as co-orbitals of the host, independently of the shape and orientation of their paths (Morais & Morbidelli 2002); in other words, to be classed as co-orbitals their orbits do not have to resemble that of the host as long as the ratio of their orbital periods equates to almost exactly one. In general, co-orbital configurations are not identified observationally but as a result of the statistical analysis of large sets of numerical integrations. There is, however, a potential exception to this standard approach; a particular type of co-orbital configuration that can be confirmed observationally, the quasi-satellite dynamical state.

Here, we study the apparent motion in host-centric equatorial coordinates of known quasi-satellites to show that they trace an analemmatic curve in the sky as observed from the host in a manner similar to that found for geosynchronous orbits. This paper is organized as follows. Section 2 discusses the so-called analemma criterion for quasi-satellites and it includes an extensive exploration of the known quasi-satellite population. The particular case of 15810 (1994 JR<sub>1</sub>), a putative accidental quasi-satellite of Pluto, is analysed in Section 3 to show that according to the analemma criterion

it is a true transient quasi-satellite of Pluto. Results are discussed in Section 4 and conclusions are summarized in Section 5.

## 2 THE ANALEMMA CRITERION

Minor bodies are confirmed as co-orbitals after statistical analysis of their simulated orbital evolution. Here, we show that there is a particular type of co-orbital configuration that may be confirmed observationally: the quasi-satellite state.

### 2.1 Quasi-satellites: a short review and a lost specimen

Minor bodies engaged in quasi-satellite behaviour with a host move near the host for the duration of the quasi-satellite episode although each pair minor-body–host is not gravitationally bound. In the Solar system and from a frame of reference centred at the Sun but corotating with the host, the quasi-satellite appears to go around the host like a regular retrograde satellite but the physical distance between the two bodies is always greater than the radius of the Hill sphere of the host (see e.g. fig. 1 in Mikkola et al. 2004). The quasi-satellite state is one of the dynamical epitomes of the 1:1 mean motion or co-orbital resonance, the other two being the Trojan or tadpole and the horseshoe resonant states (see e.g. Murray & Dermott 1999; Mikkola et al. 2006).

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Dynamical classification within the 1:1 mean motion resonance is based on the study of a critical angle, the relative mean longitude,  $\lambda_r$ , or difference between the mean longitude of the object and that of its host. If  $\lambda_r$  librates or oscillates over time, then the object under study is a co-orbital. In principle, this can only be confirmed via  $N$ -body simulations. Quasi-satellites exhibit libration of  $\lambda_r$  about  $0^\circ$  (for additional details, see e.g. Mikkola et al. 2006; de la Fuente Marcos & de la Fuente Marcos 2014, 2016a,b).

The existence of quasi-satellites was predicted more than a century ago (Jackson 1913), but the first bona fide quasi-satellite was not identified until much later —2002 VE<sub>68</sub> was confirmed as quasi-satellite of Venus by Mikkola et al. (2004). However, the first quasi-satellite may have been identified in 1973 although it was apparently lost shortly after. Using numerical integrations, Chebotarev (1974)<sup>1</sup> showed that the so-called minor planet 7617 (see his fig. 5 and table 5) was a quasi-satellite of Jupiter although he regarded this object as a distant Jovian satellite. This minor planet 7617 is clearly (see table 5 in Chebotarev 1974) not asteroid 7617 (1996 VF<sub>30</sub>), as Chebotarev (1974) followed the numbering scheme in van Houten et al. (1970). The orbital elements (1950 equinox) of the mysterious minor planet 7617 in van Houten et al. (1970) — $a = 5.0785$  au,  $e = 0.6179$ ,  $i = 4^\circ.080$ ,  $\Omega = 68^\circ.81$  and  $\omega = 209^\circ.23$ — do not match those of any known asteroid or comet; therefore, it is presumed lost.

## 2.2 Theoretical expectations

When observed from a celestial object (planetary or not) true satellites (not following synchronous orbits), co-orbitals of the Trojan or horseshoe type, and passing objects describe roughly sinusoidal paths in the sky over a sidereal orbital period. In sharp contrast, quasi-satellites appear to orbit the host when viewed in a heliocentric frame of reference that rotates with the host. As their orbital periods are very close to the sidereal period of the host, the standard sinusoidal trace becomes compressed longitudinally turning into an analemmatic curve.

The analemma or analemmatic curve —the figure-eight loop— has been traditionally linked to graphic depictions of the changing of the seasons and the equation of time (see e.g. Heath 1923; Raisz 1942; Oliver 1972; di Cicco 1979; Irvine 2001; Holbrow 2013). In addition, the trajectories of geosynchronous satellites as observed from the ground have the appearance of an analemma (Chalmers 1987). From the host, the apparent motion of a quasi-satellite during a sidereal orbital period is not too different from that of true satellites moving in synchronous orbits.

In order to show that a given quasi-satellite traces an analemmatic curve, we proceed as follows. We perform full  $N$ -body simulations in ecliptic coordinates; at time  $t$ , for a given host of coordinates,  $(x_h, y_h, z_h)$ , and axial tilt or obliquity,  $\epsilon$ , and a certain quasi-satellite located at  $(x_{qs}, y_{qs}, z_{qs})$  we can define the host-centric equatorial coordinates,  $(\alpha^*, \delta^*)$ :

$$\begin{aligned} r \cos \alpha^* \cos \delta^* &= x_{qs} - x_h \\ r \sin \alpha^* \cos \delta^* &= (y_{qs} - y_h) \cos \epsilon - (z_{qs} - z_h) \sin \epsilon \\ r \sin \delta^* &= (y_{qs} - y_h) \sin \epsilon + (z_{qs} - z_h) \cos \epsilon, \end{aligned} \quad (1)$$

where  $r$  is the distance between host and quasi-satellite at time  $t$ . For the particular case of the Earth,  $(\alpha^*, \delta^*)$  become  $(\alpha, \delta)$ , the usual geocentric equatorial coordinates. Over one sidereal period, there is

a north-south oscillation of  $\delta^*$  that is responsible for the lengthwise extension of the analemma pattern. Such libration is induced by the fact that the orbital plane of the quasi-satellite and the celestial equator at the host are, in general, tilted by a certain amount. In addition, the relative motion of a quasi-satellite with respect to its host is not uniform because, in a typical case, their orbital eccentricities are different although their semimajor axes are nearly equal and this tends to distort the analemma. The observed apparent motion results from the interplay between the two effects; when both have comparable strengths, the familiar figure-eight is obtained. In the particular case of the Earth and for an ideal quasi-satellite bright enough to be observed year-round with standard ground-based telescopes, regular astrometric observations should make it possible to plot the associated analemmatic curve without any help from numerical computations. Unfortunately, no such quasi-satellite is known to exist and  $N$ -body simulations are needed to produce synthetic astrometry to confirm the theoretical expectations.

The apparent motion of the objects studied here and plotted in Figs 1 and 2 has been computed using the Hermite scheme (Makino 1991; Aarseth 2003).<sup>2</sup> The Cartesian state vectors of the integrated bodies at the epoch 2457600.5 JD TDB (2016-July-31.0) have been retrieved from the Jet Propulsion Laboratory's (JPL) HORIZONS<sup>3</sup> system (Giorgini et al. 1996); this epoch is the  $t = 0$  instant in the simulations. Full details of these calculations can be found in de la Fuente Marcos & de la Fuente Marcos (2012b, 2014, 2016a,b).

## 2.3 The case of Venus

Asteroid 2002 VE<sub>68</sub> was confirmed as quasi-satellite of Venus by Mikkola et al. (2004). The orbital evolution of this object was further studied in de la Fuente Marcos & de la Fuente Marcos (2012b). Its orbit is quite eccentric ( $e = 0.4103$ ) and moderately inclined ( $i = 9^\circ.0070$ ). Fig. 1, top left-hand panel, shows the results of equations (1) from  $t = 0$  until 10 yr later, i.e. over 16 orbital sidereal periods of Venus. Over 16 analemmatic loops are displayed and, consistent with its significant eccentricity, one of the lobes of the analemma is very small, each loop resembling an inverted teardrop.

## 2.4 The case of the Earth

Our calculations show that our planet hosts the largest known population of quasi-satellites in the Solar system; however, their dynamical origin appears to be rather heterogeneous (de la Fuente Marcos & de la Fuente Marcos 2014, 2016a,b). There are five confirmed quasi-satellites of the Earth: 164207 (2004 GU<sub>9</sub>) (Connors et al. 2004; Mikkola et al. 2006; Wajer 2010), 277810 (2006 FV<sub>35</sub>) (Wiegert et al. 2008; Wajer 2010), 2013 LX<sub>28</sub> (Connors 2014), 2014 OL<sub>339</sub> (de la Fuente Marcos & de la Fuente Marcos 2014, 2016a) and 469219 (2016 HO<sub>3</sub>)<sup>4</sup> (de la Fuente Marcos & de la Fuente Marcos 2016b). Fig. 1, top right-hand panel, shows the apparent motion of these five objects over 10 sidereal periods; a wide range of behaviours, from a very symmetric figure-eight to very distorted teardrop shapes, is observed.

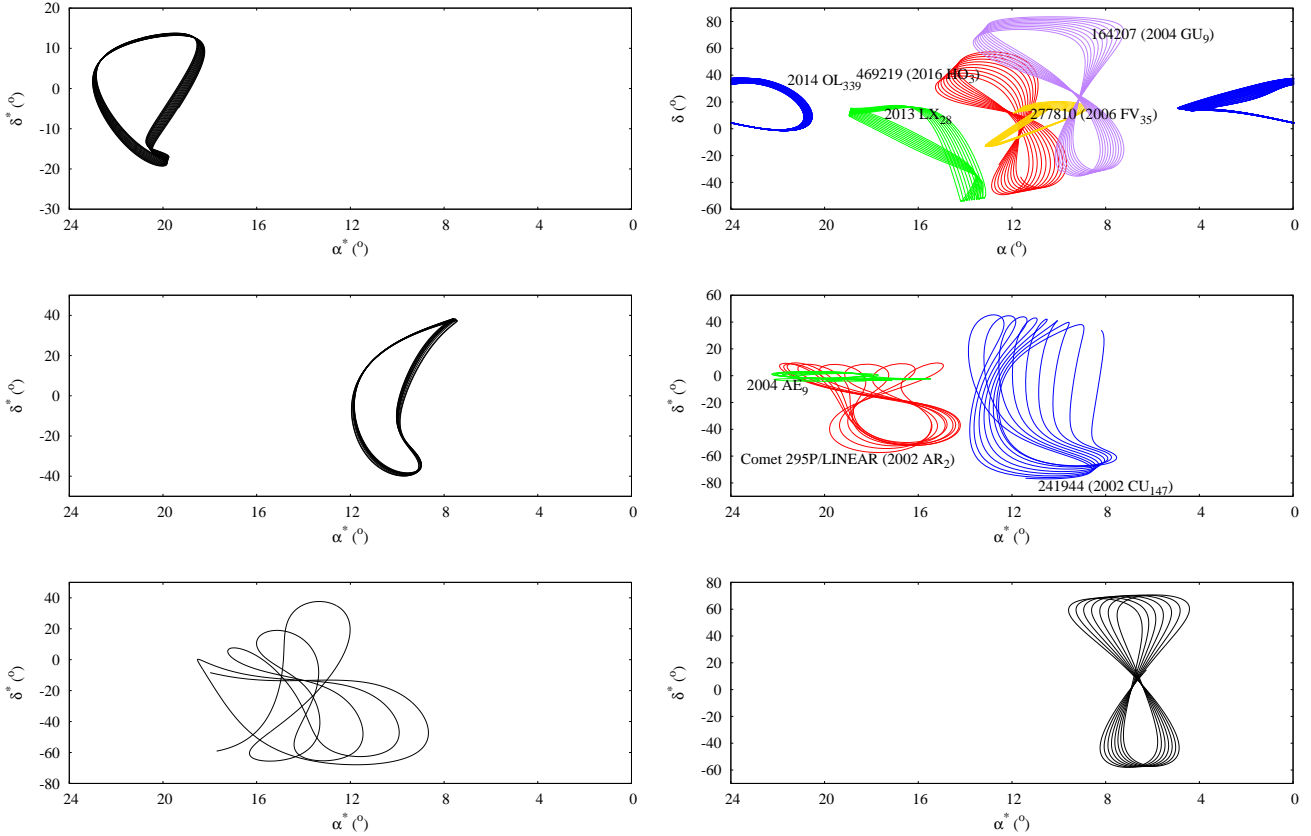
Asteroid 164207 (purple) has both moderate eccentricity ( $e = 0.1362$ ) and inclination ( $i = 13^\circ.6491$ ), and it traces a somewhat symmetric figure-eight that slowly shifts, keeping the position of

<sup>1</sup> Originally published in Russian, *Astron. Zh.*, 50, 1071-1075 (1973 September–October).

<sup>2</sup> <http://www.ast.cam.ac.uk/~sverre/web/pages/nbody.htm>

<sup>3</sup> <http://ssd.jpl.nasa.gov/?horizons>

<sup>4</sup> <http://www.jpl.nasa.gov/news/news.php?feature=6537>



**Figure 1.** Apparent motion in host-centric equatorial coordinates of known quasi-satellites. Asteroid 2002 VE<sub>68</sub> from Venus (top left-hand panel), 164207 (2004 GU<sub>9</sub>), 277810 (2006 FV<sub>35</sub>), 2013 LX<sub>28</sub>, 2014 OL<sub>339</sub> and 469219 (2016 HO<sub>3</sub>) from the Earth (top right-hand panel), 76146 (2000 EU<sub>16</sub>) from Ceres (middle left-hand panel), comet 295P/LINEAR (2002 AR<sub>2</sub>), 241944 (2002 CU<sub>147</sub>) and 2004 AE<sub>9</sub> from Jupiter (middle right-hand panel), 63252 (2001 BL<sub>41</sub>) from Saturn (bottom left-hand panel), and 309239 (2007 RW<sub>10</sub>) from Neptune (bottom right-hand panel).

the node almost fixed. Asteroid 277810 (gold) has significant eccentricity ( $e = 0.3776$ ), but low orbital inclination ( $i = 7^\circ.1041$ ); consistently, its apparent motion describes a very distorted teardrop as the effect of the eccentricity dominates. Asteroid 2013 LX<sub>28</sub> (green) follows a quite eccentric ( $e = 0.4520$ ) and very inclined ( $i = 49^\circ.9754$ ) path that translates into an apparent motion that traces an elongated teardrop-shaped curve. Asteroid 2014 OL<sub>339</sub> (blue) describes a somewhat elliptic analemma which suggests that one of the effects is nearly negligible; consistently, it follows a very eccentric ( $e = 0.4608$ ) but moderately inclined ( $i = 10^\circ.1868$ ) orbit. Finally, the orbit pursued by 469219 (red) has both low eccentricity ( $e = 0.1041$ ) and inclination ( $i = 7^\circ.77140$ ); consistently, the analemma described in the sky resembles that of 164207 with a rather symmetric shifting figure-eight.

## 2.5 The case of Ceres

Planets are not the only possible hosts of quasi-satellite bodies, dwarf planet Ceres also has one of these interesting companions (Christou 2000; Christou & Wiegert 2012), 76146 (2000 EU<sub>16</sub>). Fig. 1, middle left-hand panel, shows the results of nearly 11 sidereal periods. Asteroid 76146 follows a low-eccentricity ( $e = 0.1674$ ), low-inclination ( $i = 8^\circ.8475$ ) orbit; surprisingly, its relatively low eccentricity is high enough to induce a rather distorted teardrop shape to the resulting analemma.

## 2.6 The case of Jupiter

Jupiter is often regarded as the host of the largest known population of quasi-satellites with at least six, including asteroids and comets (Kinoshita & Nakai 2007; Wajer & Królikowska 2012). However, we failed to confirm several of the proposed candidates as present-day quasi-satellites of Jupiter. We have found three confirmed quasi-satellites of Jupiter: comet 295P/LINEAR (2002 AR<sub>2</sub>) (Kinoshita & Nakai 2007; Wajer & Królikowska 2012), 241944 (2002 CU<sub>147</sub>) (Wajer & Królikowska 2012) and 2004 AE<sub>9</sub> (Kinoshita & Nakai 2007; Wajer & Królikowska 2012). Fig. 1, middle right-hand panel, shows the results of 8.4 sidereal periods of apparent motion for these objects. Comet 295P/LINEAR (red) follows a very eccentric orbit ( $e = 0.6460$ ) that is only moderately inclined ( $i = 14^\circ.6912$ ); consistently, a rather asymmetric figure-eight is observed. This object is unlikely to be the mysterious minor planet 7617 in van Houten et al. (1970), the angular elements being too different. Asteroid 241944 (blue) pursues a relatively eccentric orbit ( $e = 0.3136$ ) that is also quite inclined ( $i = 32^\circ.8906$ ); the relatively rapidly shifting analemma exhibits somewhat symmetric lobes. In sharp contrast, 2004 AE<sub>9</sub> follows a very eccentric ( $e = 0.6459$ ) but nearly ecliptic ( $i = 1^\circ.6521$ ) orbit and its apparent motion as seen from Jupiter traces a very squashed teardrop. Again, the angular elements of 2004 AE<sub>9</sub> are very different from those of the mysterious minor planet 7617.

### 2.7 The case of Saturn

Gallardo (2006) indicated that 15504 (1999 RG<sub>33</sub>) could be a quasi-satellite of Saturn. Our calculations show that it is indeed a transient co-orbital of Saturn, but not a quasi-satellite like the objects previously discussed. Its apparent motion somewhat resembles that of Molniya-type artificial satellites of the Earth (the apocentre of the very eccentric orbits occurs at a large declination). However, 63252 (2001 BL<sub>41</sub>) that was discovered by Gehrels et al. (2001) is currently a short-lived quasi-satellite of Saturn that will change its current dynamical status in about 130 yr from now. Asteroid 63252 follows an eccentric ( $e = 0.2948$ ) but moderately inclined ( $i = 12.5163^\circ$ ) orbit. Fig. 1, bottom left-hand panel, shows the apparent motion of this object as seen from Saturn for about 4.4 sidereal periods, prior to leaving its current quasi-satellite state. The analemmatic loops described by this object are very distorted as a result of its unstable dynamical behaviour and the lobes have somewhat different sizes because the effect derived from eccentricity is stronger than that of inclination. Although its orbital evolution is rather chaotic, 63252—an organic rich D-type asteroid (Doresoundiram et al. 2003)—has been pre-selected by NASA for an in situ exploration mission (Ryan et al. 2009).

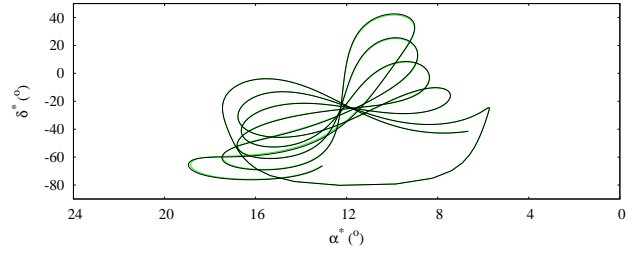
### 2.8 The case of Neptune

Asteroid 309239 (2007 RW<sub>10</sub>) is so far the only confirmed quasi-satellite of Neptune (de la Fuente Marcos & de la Fuente Marcos 2012a) and it is one of the largest known co-orbital companions in the Solar system with a diameter of about 250 km. It follows an eccentric ( $e = 0.3004$ ) and rather inclined ( $i = 36.1755^\circ$ ) orbit. Fig. 1, bottom right-hand panel, shows 7.3 sidereal periods of a very regular analemma of the figure-eight type with both lobes of nearly the same size which confirms that the effects derived from eccentricity and inclination have very similar strength in this case.

## 3 ACCIDENTAL QUASI-SATELLITES: THE CASE OF PLUTINO 15810 (1994 JR<sub>1</sub>)

Quasi-satellites are not exclusive of planetary hosts as the case of 76146 (2000 EU<sub>16</sub>) and Ceres confirms. Yu & Tremaine (1999) and Tiscareno & Malhotra (2009) used numerical simulations to predict the existence of minor bodies experiencing quasi-satellite behaviour with respect to dwarf planet Pluto. Plutino 15810 (1994 JR<sub>1</sub>) was identified as an accidental quasi-satellite of Pluto by de la Fuente Marcos & de la Fuente Marcos (2012c). It was termed accidental because, for this object,  $\lambda_r$  circulates with a superimposed libration resulting from the oscillation of the orbital period induced by the 2:3 mean motion resonance with Neptune. Such libration plays a role in triggering and terminating the quasi-satellite phase. Porter et al. (2016) have used astrometry acquired by NASA's New Horizons spacecraft to improve the already robust orbital solution available for this object (see Appendix A) and revisit its quasi-satellite status. The new data have been used to argue that the quasi-satellite nature of 15810 must be rejected.<sup>5</sup>

Fig. 2 clearly shows that although the orbital solution of 15810 has been indeed greatly improved using New Horizons data (see Appendix A), its orbital evolution still matches the one described in de la Fuente Marcos & de la Fuente Marcos (2012c). In black,



**Figure 2.** Apparent motion in Pluto-centric equatorial coordinates of 15810 (1994 JR<sub>1</sub>). The black curve shows the shifting analemma resulting from the dynamical evolution of the latest version of the orbit that includes data from NASA's New Horizons spacecraft (third orbital solution in Table A1). The green curve is equivalent to the black one but using the second orbital solution in Table A1.

we have the apparent motion resulting from the latest orbit available for 15810 (third orbital solution in Table A1). The figure displays the time interval of interest—the one showing analemmatic behaviour—that goes from 1200 years prior to  $t = 0$  to 200 years afterwards or about 5.6 sidereal orbital periods of Pluto. The analemma shifts rapidly and it is quite distorted because the orbits of both Pluto and 15810 are eccentric and there is a chaotic interaction between the two bodies. Within the context of the analemma criterion, the behaviour observed in Fig. 2 is not very different from that of some of the objects in Fig. 1. The apparent motion of 15810 closely resembles that of comet 295P/LINEAR (2002 AR<sub>2</sub>) or 63252 (2001 BL<sub>41</sub>). If the pre-NASA's New Horizons distant encounter orbit (second orbital solution in Table A1) is used (green curve), the differences are minimal. Porter et al. (2016) argue that, for 15810, instead of transient quasi-satellite behaviour we should speak of periodic (every 2.4 Myr) scattering conjunctions; however, Fig. 2 suggests that 15810 is not different from comet 295P or 63252 in dynamical terms when the analemma criterion is applied during one of its encounters with Pluto. Therefore, Pluto has at least one present-day (transient and recurrent) quasi-satellite, 15810.

## 4 DISCUSSION

Our analysis so far argues in favour of a conjecture: objects that follow a quasi-satellite path with respect to a host trace an analemmatic curve in the sky as observed from the host over a sidereal orbital period. Conversely, any object tracing an analemmatic curve in the sky as observed from the host over a sidereal year must be a quasi-satellite of the host. Unfortunately, a rigorous mathematical or even a numerical proof of the assumed theorem and its inverse is out of the scope of this work because (1) a complete theory of quasi-satellite motion still remains elusive (see e.g. Mikkola et al. 2006) and (2) the relevant volume of the orbital parameter space to be explored is simply too large. Intuitively, the truthfulness of our conjecture can hardly be argued. The position of the quasi-satellite as seen from the host is subjected to two periodic librations. At some point during the sidereal year, the quasi-satellite is east of the host, very nearly half an orbital period later it is west from the host. As the quasi-satellite bean-shaped loop (see e.g. fig. 1 in Mikkola et al. 2004) drifts back and forth, the peri-host shifts from eastwards to westwards from the host (this causes the loop drift). This behaviour is mainly the result of the difference in eccentricity between host and quasi-satellite and drives the oscillation in host-centric right ascension. The relative inclination between the equa-

<sup>5</sup> <http://www.nasa.gov/feature/new-horizons-collects-first-science-on-a-post-pluto-object>



torial plane of the host and the orbital plane of the quasi-satellite drives the oscillation in host-centric declination. These two oscillations have (nearly) commensurable frequencies because host and quasi-satellite have very similar orbital periods and generate the analemma.

Co-orbitals have been traditionally classified as such after the statistical analysis of numerical simulations. However, an algorithm to decide whether an object is a quasi-satellite of a given host, not based on  $N$ -body simulations, is described in detail in section 4 of Mikkola et al. (2006). The analemmas or analemata in Figs 1 and 2 show that, in the particular case of quasi-satellites, astrometry can be readily used to perform a reliable classification. Figs 1 and 2 also show that both very regular —164207 (2004 GU<sub>6</sub>), 469219 (2016 HO<sub>3</sub>) or 309239 (2007 RW<sub>10</sub>)— and rather irregular —comet 295P/LINEAR (2002 AR<sub>2</sub>), 63252 (2001 BL<sub>41</sub>) or 15810 (1994 JR<sub>1</sub>)— short-term evolutions are possible. The analemma traced by the quasi-satellite encodes relevant orbital information. Distorted, rapidly shifting analemmas are characteristic of quasi-satellites moving in strongly perturbed orbits.

Our calculations show that, if a suitable quasi-satellite is found, it can be used as a permanent platform to install instrumentation that may be used to monitor permanently the host body and enable a relatively stable communications relay for subsequent missions (e. g. landing quasi-autonomous vehicles on the host) at zero fuel cost because the quasi-satellite behaves like a geosynchronous satellite from the point of view of the host body. For this task, the smaller its average distance from the host the better (see e.g. the case of 469219 as discussed in de la Fuente Marcos & de la Fuente Marcos 2016b). Artificial quasi-satellites are also possible (see e.g. Kogan 1989 for the Phobos mission). For quasi-satellites sufficiently close to a host, substantial parallax may occur; therefore, and depending on the location of the observer on the surface of the host, different analemmas may be observed. This issue together with the shift of the analemma loop induced by orbital evolution requires robotic tracking of the quasi-satellite yearly movement around its analemma. The use of quasi-satellite trajectories in astrodynamics has been frequently discussed (see e.g. Kogan 1989; Lidov & Vashkov'yak 1993, 1994; Mikkola & Prioroc 2016). It could be the case that the analemmatic behaviour described here had been found before within the context of astrodynamical studies (see e.g. Kogan 1989).

## 5 CONCLUSIONS

In this paper, we have explored a new criterion to identify quasi-satellites. In sharp contrast with the numerical strategies customarily applied in the study of co-orbital bodies, the criterion described here can make direct use of observational astrometric data. Our conclusions can be summarized as follows.

- (i) Bona fide quasi-satellites trace paths in the sky which repeat every sidereal period when observed from their hosts. These paths can be described as analemmatic curves similar to those found for geosynchronous orbits. The analemma shifts as the orbit of the quasi-satellite changes over time.
- (ii) The existence of this analemmatic behaviour turns quasi-satellites, natural or artificial, into potentially interesting platforms for the future of space exploration.
- (iii) The Earth has the largest known number of present-day quasi-satellites, five. Jupiter comes in second place with three. Venus, Saturn, Neptune and dwarf planet Ceres have one each.

- (iv) Applying the analemma criterion, Plutino 15810 (1994 JR<sub>1</sub>) is as good a quasi-satellite as it may get. Therefore, dwarf planet Pluto hosts at least one quasi-satellite at present.
- (v) Asteroid 63252 (2001 BL<sub>41</sub>) is a present-day transient quasi-satellite of Saturn.
- (vi) Historically, the first object identified as quasi-satellite (in this case of Jupiter) was an asteroid moving in a comet-like orbit. Unfortunately, this object appears to have been lost since its announcement back in 1973.

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## REFERENCES

- Aarseth S. J., 2003, Gravitational N-body simulations. Cambridge Univ. Press, Cambridge, p. 27
- Chalmers J. S., 1987, *Am. J. Phys.*, 55, 548
- Chebotaev G. A., 1974, *SvA*, 17, 677
- Christou A. A., 2000, *A&A*, 356, L71
- Christou A. A., Wiegert P., 2012, *Icarus*, 217, 27
- Connors M., 2014, *MNRAS*, 437, L85
- Connors M., Veillet C., Brasser R., Wiegert P., Chodas P., Mikkola S., Innanen K., 2004, *Meteoritics Planet. Sci.*, 39, 1251
- de la Fuente Marcos C., de la Fuente Marcos R., 2012a, *A&A*, 545, L9
- de la Fuente Marcos C., de la Fuente Marcos R., 2012b, *MNRAS*, 427, 728
- de la Fuente Marcos C., de la Fuente Marcos R., 2012c, *MNRAS*, 427, L85
- de la Fuente Marcos C., de la Fuente Marcos R., 2014, *MNRAS*, 445, 2961
- de la Fuente Marcos C., de la Fuente Marcos R., 2016a, *MNRAS*, 455, 4030
- de la Fuente Marcos C., de la Fuente Marcos R., 2016b, *MNRAS*, in press (arXiv:1608.01518)
- di Cicco D., 1979, *Sky Telescope*, 57, 536
- Dorressoundiram A., Tozzi G. P., Barucci M. A., Boehnhardt H., Fornasier S., Romon J., 2003, *AJ*, 125, 2721
- Gallardo T., 2006, *Icarus*, 184, 29
- Gehrels T., Gleason A. E., McMillan R. S., Montani J. L., Larsen J. A., Marsden B. G., 2001, *MPEC Circ.*, MPEC 2001-B44
- Giorgini J. D. et al., 1996, *BAAS*, 28, 1158
- Heath W., 1923, *The Observatory*, 46, 286
- Holbrow C. H., 2013, e-print arXiv:1302.0765
- Irvine S., 2001, *J. R. Astron. Soc. Can.*, 95, 273
- Jackson J., 1913, *MNRAS*, 74, 62
- Kinoshita H., Nakai H., 2007, *Celest. Mech. Dyn. Astron.*, 98, 181
- Kogan A. Y., 1989, *Cosmic Res.*, 26, 705
- Lidov M. L., Vashkov'yak M. A., 1993, *Cosmic Res.*, 31, 187
- Lidov M. L., Vashkov'yak M. A., 1994, *Astron. Lett.*, 20, 188
- Makino J., 1991, *ApJ*, 369, 200
- Mikkola S., Brasser R., Wiegert P., Innanen K., 2004, *MNRAS*, 351, L63
- Mikkola S., Innanen K., Wiegert P., Connors M., Brasser R., 2006, *MNRAS*, 369, 15
- Mikkola S., Prioroc C.-L., 2016, *MNRAS*, 457, 1137
- Morais M. H. M., Morbidelli A., 2002, *Icarus*, 160, 1
- Murray C. D., Dermott S. F., 1999, *Solar System Dynamics*, Cambridge Univ. Press, Cambridge, p. 97
- Oliver B. M., 1972, *Sky Telescope*, 44, 20
- Porter S. B. et al., 2016, *ApJL*, submitted (arXiv:1605.05376)
- Raisz E., 1942, *Sky Telescope*, 1, 11
- Ryan E. L. et al., 2009, *Am. Astron. Soc. – DPS meeting*, 41, 16.26

- Tiscareno M. S., Malhotra R., 2009, *AJ*, 138, 827  
 van Houten C. J., van Houten-Groeneveld I., Herget P., Gehrels T., 1970, *A&AS*, 2, 339  
 Wajer P., 2010, *Icarus*, 209, 488  
 Wajer P., Królikowska M., 2012, *Acta Astron.*, 62, 113  
 Wiegert P. A., DeBoer R., Brasser R., Connors M., 2008, *J. R. Astron. Soc. Can.*, 102, 52  
 Yu Q., Tremaine S., 1999, *AJ*, 118, 1873

## APPENDIX A: COMPARISON BETWEEN THE PRE- AND POST-NEW HORIZONS DATA ORBITAL SOLUTIONS

Table A1 shows three orbital solutions for 15810 (1994 JR<sub>1</sub>). The third column corresponds to the one currently available and includes astrometry acquired by NASA’s New Horizons spacecraft and discussed in Porter et al. (2016). As pointed out by Porter et al. (2016), the New Horizons data have improved the orbital solution of 15810 very significantly. However, numerical simulations equivalent to those in de la Fuente Marcos & de la Fuente Marcos (2012c) but making use of the second and third orbital solutions in Table A1 still produce the same basic results; the differences are simply too small to claim any dramatic change in the nature of the orbital evolution of 15810 as a result of the new and indeed improved orbit. The original description in de la Fuente Marcos & de la Fuente Marcos (2012c) is certainly still valid. Nevertheless, the astrometry discussed in Porter et al. (2016) confirms beyond any doubt the role that NASA’s New Horizons spacecraft may play in improving the orbital solutions of many trans-Neptunian objects over the next decade or so.

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**Table A1.** Heliocentric Keplerian orbital elements of 15810 (1994 JR<sub>1</sub>) from JPL’s Small-Body Database and HORIZONS On-Line Ephemeris System; values include the  $1\sigma$  uncertainty. The orbit in the left-hand column was the one available back in 2012 and it was used by de la Fuente Marcos & de la Fuente Marcos (2012c); this orbit is referred to the epoch 2456200.5 JD CT (2012-September-30.0) and it was computed using 43 observations with an arc-length of 2236 d. The orbital solution in the column next to it was computed on 2015 October 05 13:55:21 UT and it is referred to the epoch 2457600.5 JD TDB (2016-July-31.0) TDB; it was computed using 49 observations with an arc-length of 7701 d. The third orbit is the one currently available and it was computed on 2016 June 21 15:49:21 UT. This new and improved orbital solution includes astrometry acquired by NASA’s New Horizons spacecraft and is referred to the same epoch as the previous one; it was computed using 78 observations with an arc-length of 8002 d.

Semimajor axis, $a$ (au)	=	39.24±0.02	39.427±0.011	39.4224±0.0009
Eccentricity, $e$	=	0.1143±0.0003	0.1196±0.0002	0.119501±0.000010
Inclination, $i$ (°)	=	3.8032±0.0002	3.80801±0.00005	3.80802±0.00005
Longitude of the ascending node, $\Omega$ (°)	=	144.753±0.011	144.6859±0.0011	144.6854±0.0007
Argument of perihelion, $\omega$ (°)	=	102.1±0.2	101.55±0.03	101.535±0.012